Summary

Tuesday, January 29, 2013 1:53 PM

Moving charge(s) in magnetic field experiences force

- one charged particle : $\vec{F}_B = q\vec{v}\times\vec{B}$ ⇒ Circular motion
  $|q|vB = m\frac{v^2}{r}$ ⇒ $r = \frac{mv}{|q|B}$ ⇒ $T = \frac{2\pi r}{v}$, $f = \frac{1}{T}$
  → centripetal acceleration (a)
  → centripetal force (F = ma)
- current : $\vec{F}_B = i\vec{L}\times\vec{B}$

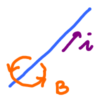
Force between two long parallel wires carrying currents

$F = \frac{\mu_0 L i_a i_b}{2\pi d}$

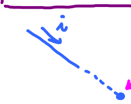
direction
parallel currents : attract
antiparallel currents : repel

Current produces magnetic field

- current-length element ($i\,d\vec{s}$) : $d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$ ← Biot - Savart Law
- long straight wire : $B = \frac{\mu_0 i}{2\pi R}$
  R → perpendicular distance
  (use right-hand rule to find the direction of B.)



- radial segment (current directly toward or away) to or from a point does not create any magnetic field there

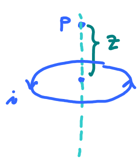


contribution from the radial segment = 0 at this point

- at the center of circular arc $B = \frac{\mu_0 i}{2R}\times\left(\frac{\phi}{2\pi}\right) = \frac{\mu_0 i}{2R}\left(\frac{\phi}{360°}\right)$
  ← in radians, ← in degrees
  fraction of the complete circle

complete circle ↓ $B = \frac{\mu_0 i}{2R}$ ↑ $z = 0$

- coil (circular loop) : $B = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}$



- Inside a solenoid $B = \mu_0 i n$ ← n: # turns per unit length

} magnetic dipole

Ampere's law : $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$

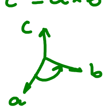
- use superposition when the wire (loop) is a combination of the shapes above.

Right-hand rule $\vec{c} = \vec{a}\times\vec{b}$

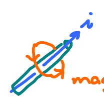
$\vec{F} = q\vec{v}\times\vec{B}$

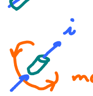
$\vec{F} = i\vec{L}\times\vec{B}$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$



Curled-straight right-hand rule





magnetic field at center of the arc

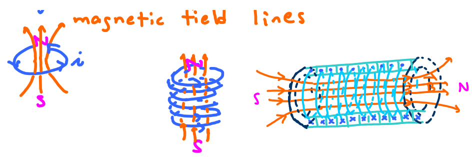
current in an arc

current in a loop or solenoid

magnetic field inside

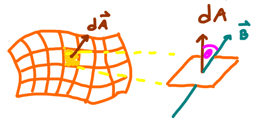
magnetic dipole

magnetic field lines



Induction: (time-varying) Change in magnetic flux induces emf and current
$\mathcal{E}_{ind}$ [V] $\quad i_{ind}$ [A]

$$\underset{[Wb]}{\Phi_B} = \int d\vec{B} = \int \underbrace{\vec{B}\cdot d\vec{A}}$$
$= B\,dA\cos\theta$
$= B_\perp dA = B\,dA_\perp$



$= BA\cos\theta$ for uniform magnetic field through flat area A

$\Phi_B(t) = B(t)\,A(t)\cos(\theta(t))$

time-varying magnetic flux

Ohm's law ($\mathcal{E} = iR$)

previous chapter (Biot-Savart Law and its special cases)

$$\Delta\Phi_B \rightarrow \mathcal{E}_{ind} \rightarrow i_{ind} \;(\rightarrow B_{ind})$$

① Use Faraday's law to determine the magnitude of $\mathcal{E}_{ind}$

Faraday's law of induction:
$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt}$$

② Use Lenz's law to determine the direction of $\mathcal{E}_{ind}$, i_{ind}, and B_{ind}.

Lenz's law:
"The direction of any induction effect is that which opposes the $\underbrace{\text{cause}}_{\Delta\Phi_B}$ of the effect."

Recipe for solving induction problem

① Find the expression for the magnetic flux Φ_B through the coil/loop.

$$\Phi_B = \int \vec{B}\cdot d\vec{A} \underset{\substack{\text{uniform } \vec{B} \\ \text{and flat area}}}{=} BA\cos\theta \underset{\substack{\text{uniform } \vec{B} \\ \perp \text{ flat area}}}{=} BA$$

② Find the slope $\frac{d\Phi_B}{dt}$

③ Apply Faraday's law $\quad \mathcal{E}_{ind} = -\frac{d\Phi_B}{dt}N$

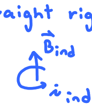
Multiply by N for the coil that has N turns

Magnitude of the induced emf is $|\mathcal{E}_{ind}| = \left|\frac{d\Phi_B}{dt}\right| N$

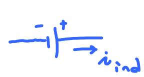
④ Find the direction of $\underbrace{\text{the change in } \Phi_B}_{\Delta\Phi_B}$.

By Lenz's law, $\vec{B}_{ind}$ should be in the direction that opposes $\Delta\Phi_B$.
(In many problems, $\vec{B}$ is the only quantity that changes. In such problems, $\vec{B}_{ind}$ should be in the direction that opposes $\Delta\vec{B}$.)

⑤ Direction of i_{ind} is given by the curled-straight right-hand rule.



Direction of $\mathcal{E}_{ind}$ is such that i_{ind} flows out of its positive terminal.

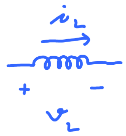


⑥ Magnitude of $i_{ind} \underset{\text{Ohm's law}}{=} \frac{\mathcal{E}_{ind}}{R}$.

R

Ohm's law

Inductor .



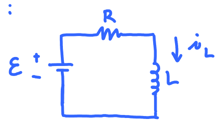
$$v_L = L\frac{di_L}{dt}$$

inductance $\equiv \dfrac{N\Phi_B}{i} = n^2\mu_0 \ell A$

[H : henry]

The current through an inductor can not jump (change) abruptly.

RL circuit :



$$i_L(t) = e^{-\frac{R}{L}(t-t_0)}(i_0 - i_\infty) + i_\infty$$

$i_0 = i(t)$ at time $t = t_0$

$i_\infty = \dfrac{\mathcal{E}}{R}$ ← In the long-run, the inductor acts like ordinary connecting wire.

Key ideas for solving RL circuit w/ switch(es) (may have multiple resistors/inductors.)

① When there is no change in the circuit configuration for a long time, the inductor acts like a short circuit (ordinary connecting wire).

② When there is some change in the circuit (e.g. SW is open or closed), the current through the inductor cannot change abruptly

Therefore, immediately after the change occurs, the current through the inductor must be the same as its value just before the change happens.

(Steady-state) Analysis for ac

Sinusoid : $x(t) = X_m \cos(\omega t + \phi) = X_m \cos(2\pi f t + \phi)$ (Amplitude, angular freq., phase, frequency)

In standard form, $X_m \geq 0$ and $-180° \leq \phi \leq 180°$.

Conversion to standard form : $\sin\alpha = \cos(\alpha - 90°)$

$-\cos\alpha = \cos(\alpha \pm 180°)$

R, L, C in ac circuit (driven by ac source)

	R	L	C
	$v = Ri$	$v = L\frac{di}{dt}$	$i = C\frac{dv}{dt}$
Reactance	–	ωL inductive reactance	capacitive reactance $\frac{1}{\omega C}$
Resistance	R	–	–
$V_m =$	$R I_m$	$(\omega L) I_m$	$(1/\omega C) I_m$
$\phi_v =$	ϕ_i	$\phi_i + 90°$	$\phi_i - 90°$

Gauss's law for magnetic fields :

$$\Phi_B = \oint \vec{B}\cdot d\vec{A} = 0$$

Integration is taken over a closed Gaussian surface

⇒ Magnetic monopoles do not exist.

(There can be no net magnetic flux through the surface because there can be no net "magnetic charge" enclosed by the surface.

single magnetic pole

Electromagnetic (EM) Wave

We consider EM waves whose $\vec{E}$ and $\vec{B}$ vary sinusoidally

Important characteristics :

- $\vec{E}$ and $\vec{B}$ have the same frequency
- $\vec{E}$ and $\vec{B}$ are in-phase
- $\vec{E}$ and $\vec{B}$ are perpendicular
- $\vec{E} \times \vec{B}$ gives the direction in which the wave travels.

Amplitude of $\vec{E}$ field

Wave speed $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = \dfrac{E_m}{B_m} = \dfrac{E}{B} = 299,792,458 \text{ m/s} = f\lambda$

Amplitude of $\vec{B}$ field